AP Calculus AB | Activity 0123 | Name:
Dr. Paul L. Bailey | Thursday, January 23, 2025

Problem 1. Let A be the set of states in the United States, and let B be the set of all cities in the United States. Let $f : A \to B$ be given by mapping a state to its capital city.

(a) Is f injective? Is f surjective?

(b) Let $C = \{\texttt{AZ}, \texttt{CA}, \texttt{UT}\}$. Find $f(C)$.

(c) Let D be the set of the ten largest cities in the united states. Find $f^{-1}(D)$.

Problem 2. Let A be the set of all presidents of the United States, and let B be the set of all years between 1776 and 2023. Let $f : A \to B$ be given by $f : \texttt{president} \mapsto \texttt{inauguration year}$.

(a) Is f injective? Is f surjective? Justify your answer.

(b) Let C denote the set of presidents whose last names begins with the letter `B`. Find C and $f(C)$.

(c) Let $D = [1860, 1880] \cap B$. Find D and $f^{-1}(D)$.

Problem 3. Let $\mathbb{N}$ be the set of natural numbers. Let $A = [50, 70] \cap \mathbb{N}$. Define a function $f : \mathbb{N} \to \mathbb{N}$ by $f(n) = 3n$. Note that A is in both the domain and the codomain of f.

(a) Find the image $f(A)$.

(b) Find the preimage $f^{-1}(A)$.

(c) Is f injective? Is f surjective?

Problem 4. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^3 - 3x^2 - 2x + 11$. Find $f^{-1}(5)$.

Problem 5. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^3 - 6x^2 + 11x - 3$. Find $f^{-1}(3)$.